

FINAL JEE(Advanced) EXAMINATION - 2020

(Held On Sunday 27th SEPTEMBER, 2020)

PAPER-1

TEST PAPER WITH SOLUTIONS

PART-1 : PHYSICS

SECTION-1 : (Maximum Marks : 18)

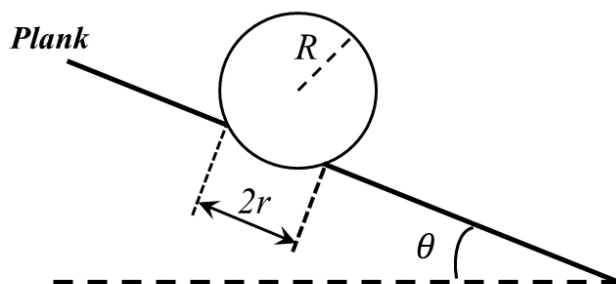
- This section contains **SIX (06)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

1. A football of radius R is kept on a hole of radius r ($r < R$) made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale) -



(A) $\sin \theta = \frac{r}{R}$

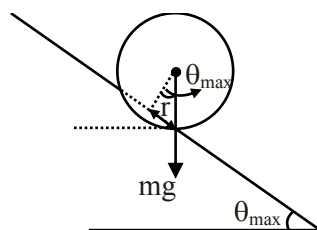
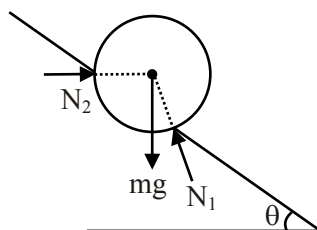
(B) $\tan \theta = \frac{r}{R}$

(C) $\sin \theta = \frac{r}{2R}$

(D) $\cos \theta = \frac{r}{2R}$

Ans. (A)

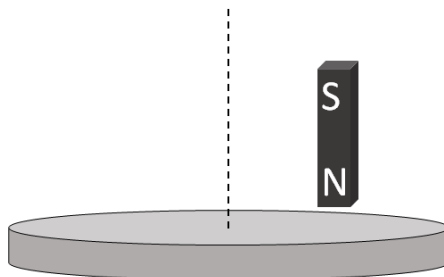
Sol.



For θ_{\max} , the football is about to roll, then $N_2 = 0$ and all the forces (Mg and N_1) must pass through contact point

$$\therefore \cos(90^\circ - \theta_{\max}) = \frac{r}{R} \Rightarrow \sin \theta_{\max} = \frac{r}{R}$$

2. A light disc made of aluminium (a nonmagnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (figure is schematic and not drawn to scale)-



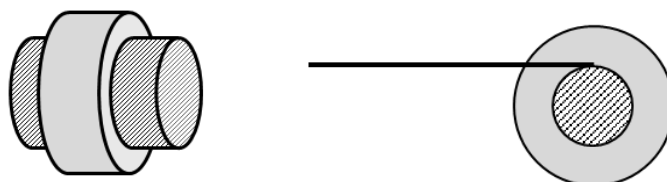
- (A) rotate in the direction opposite to the direction of magnet's motion
- (B) rotate in the same direction as the direction of magnet's motion
- (C) not rotate and its temperature will remain unchanged
- (D) not rotate but its temperature will slowly rise

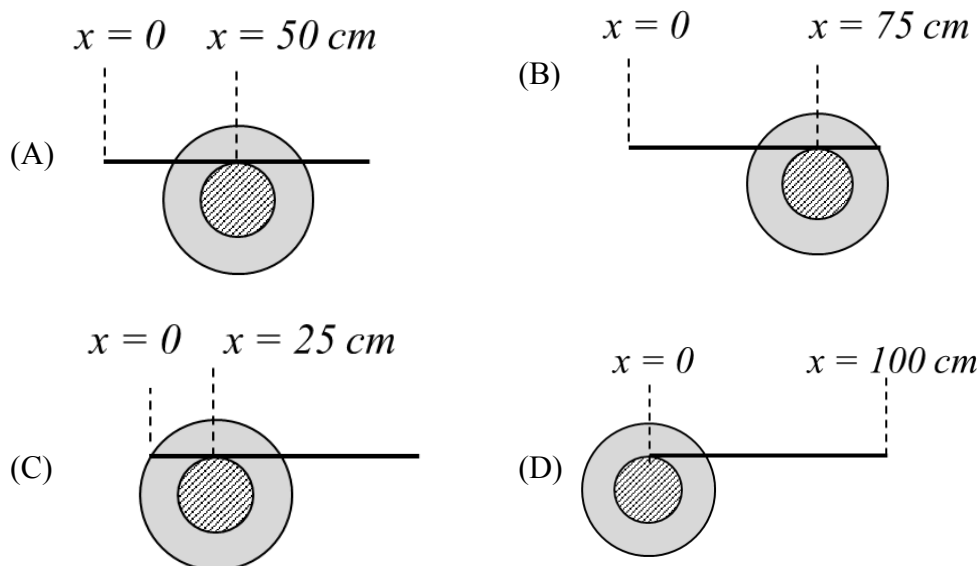
Ans. (B)

Sol. When the magnet is moved, it creates a state where the plate moves through the magnetic flux, due to which an electromotive force is generated in the plate and eddy currents are induced. These currents are such that it opposes the relative motion \Rightarrow disc will rotate in the direction of rotation of magnet.

Note : This apparatus is called Arago's disk.

3. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (figures are schematic and not drawn to scale)-





Ans. (B)

Sol. For no slipping at the ground,

$$V_{\text{centre}} = \omega R \quad (R \text{ is radius of roller})$$

$$\therefore \text{Velocity of scale} = (V_{\text{centre}} + \omega r) \quad [r \text{ is radius of axle}]$$

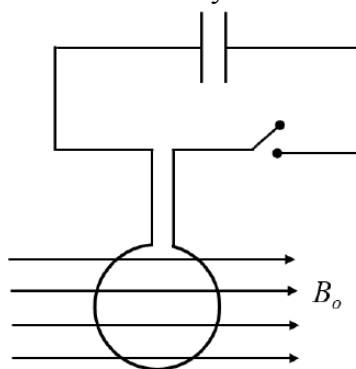
$$\text{Given, } V_{\text{centre}} \cdot t = 50 \text{ cm}$$

$$\therefore \text{Distance moved by scale} = (V_{\text{centre}} + \omega r)t$$

$$= \left(V_{\text{centre}} + \frac{V_{\text{centre}} r}{R} \right) t = \frac{3 V_{\text{centre}}}{2} \cdot t = 75 \text{ cm}$$

Therefore relative displacement (with respect to centre of roller) is $(75 - 50) \text{ cm} = 25 \text{ cm}$.

4. A circular coil of radius R and N turns has negligible resistance. As shown in the schematic figure, its two ends are connected to two wires and it is hanging by those wires with its plane being vertical. The wires are connected to a capacitor with charge Q through a switch. The coil is in a horizontal uniform magnetic field B_0 parallel to the plane of the coil. When the switch is closed, the capacitor gets discharged through the coil in a very short time. By the time the capacitor is discharged fully, magnitude of the angular momentum gained by the coil will be (assume that the discharge time is so short that the coil has hardly rotated during this time)-



(A) $\frac{\pi}{2} N Q B_0 R^2$

(B) $\pi N Q B_0 R^2$

(C) $2\pi N Q B_0 R^2$

(D) $4\pi N Q B_0 R^2$

Ans. (B)

Sol. Torque experienced by circular loop = $\vec{M} \times \vec{B}$

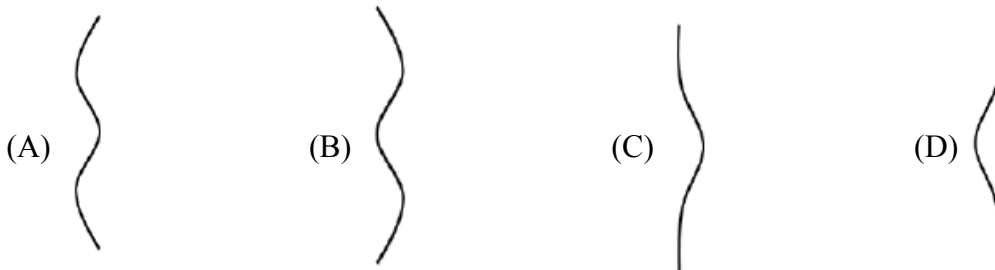
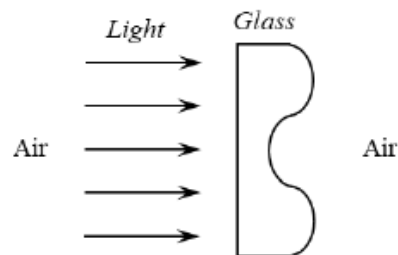
where \vec{M} is magnetic moment

\vec{B} is magnetic field

$$\therefore \tau = i\pi R^2 N B_0 \text{ [at the instant shown } \theta = \pi/2]$$

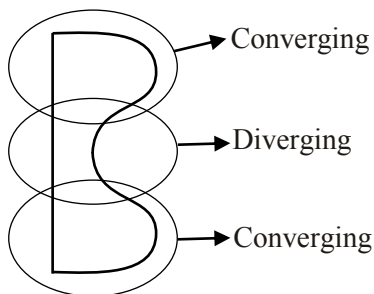
$$\therefore \tau dt = d\vec{L} = i\pi R^2 N B_0 dt = Q\pi R^2 N B_0 [idt = Q]$$

5. A parallel beam of light strikes a piece of transparent glass having cross section as shown in the figure below. Correct shape of the emergent wavefront will be (figures are schematic and not drawn to scale)-

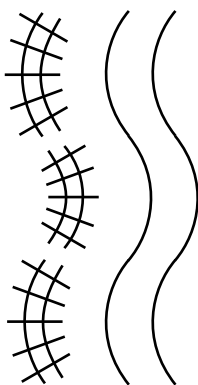


Ans. (A)

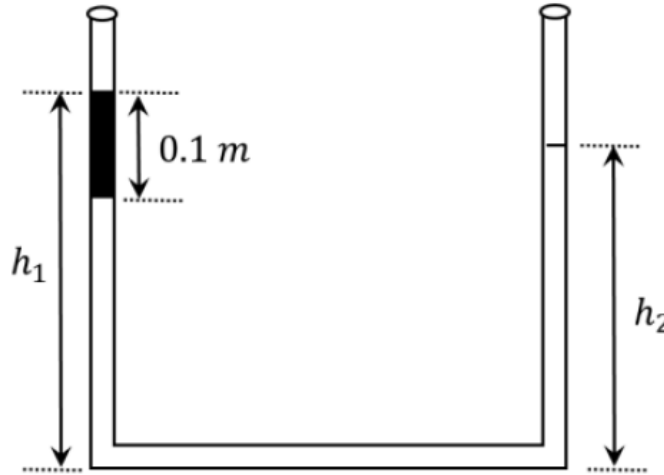
Sol.



(A)



6. An open-ended U-tube of uniform cross-sectional area contains water (density 10^3 kg m^{-3}). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is-



- (A) $\frac{15}{14}$ (B) $\frac{35}{33}$ (C) $\frac{7}{6}$ (D) $\frac{5}{4}$

Ans. (B)

Sol. $h_1 + h_2 = 0.29 \times 2 + 0.1$

$$h_1 + h_2 = 0.68 \quad \dots\dots(1)$$

$$\Rightarrow P_0 + \rho_k g(0.1) + \rho_w g(h_1 - 0.1) \quad [\rho_k = \text{density of kerosene} \ \& \ \rho_w = \text{density of water}]$$

$$-\rho_w g h_2 = P_0$$

$$\Rightarrow \rho_k g(0.1) + \rho_w g h_1 - \rho_w g \times (0.1)$$

$$= \rho_w g h_2$$

$$\Rightarrow 800 \times 10 \times 0.1 + 1000 \times 10 \times h_1$$

$$- 1000 \times 10 \times 0.1 = 1000 \times 10 \times h_2$$

$$\Rightarrow 10000 (h_1 - h_2) = 200$$

$$\Rightarrow h_1 - h_2 = 0.02 \quad \dots\dots(2)$$

$$\Rightarrow \boxed{h_1 = 0.35}$$

$$\Rightarrow \boxed{h_2 = 0.33}$$

So, $\boxed{\frac{h_1}{h_2} = \frac{35}{33}}$

SECTION-2 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chose (i.e. the question is unanswered);
Negative Marks	: -2	In all other cases

7. A particle of mass m moves in circular orbits with potential energy $V(r)=Fr$, where F is a positive constant and r is its distance from the origin. Its energies are calculated using the Bohr model. If the radius of the particle's orbit is denoted by R and its speed and energy are denoted by v and E , respectively, then for the n^{th} orbit (here h is the Planck's constant)-

(A) $R \propto n^{1/3}$ and $v \propto n^{2/3}$

(B) $R \propto n^{2/3}$ and $v \propto n^{1/3}$

(C) $E = \frac{3}{2} \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3}$

(D) $E = 2 \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3}$

Ans. (B,C)

Sol. $U = Fr$

[Using U = Potential energy and v = velocity, to avoid confusion between their symbols]

$$\Rightarrow \text{Force} = \frac{-dU}{dr} = -F$$

$$\Rightarrow \text{Magnitude of force} = \text{Constant} = F$$

$$\Rightarrow F = \frac{mv^2}{R} \quad \dots\dots(1)$$

$$\Rightarrow mvR = \frac{nh}{2\pi} \quad \dots\dots(2)$$

$$\Rightarrow F = \frac{m}{R} \times \frac{n^2 h^2}{4\pi^2} \times \frac{1}{m^2 R^2}$$

$$\Rightarrow R = \left(\frac{n^2 h^2}{4\pi^2 m F} \right)^{1/3} \quad \dots\dots(3)$$

$$\Rightarrow v = \frac{nh}{2\pi m R}$$

$$\Rightarrow v = \frac{nh}{2\pi m} \left(\frac{4\pi^2 mF}{n^2 h^2} \right)^{1/3}$$

$$\Rightarrow v = \frac{n^{1/3} h^{1/3} F^{1/3}}{2^{1/3} \pi^{1/3} m^{2/3}} \dots\dots(4)$$

(B) is correct

$$\Rightarrow E = \frac{1}{2}mv^2 + U$$

$$= \frac{1}{2}mv^2 + FR$$

$$\Rightarrow E = \frac{1}{2}m \left(\frac{n^{2/3} h^{2/3} F^{2/3}}{2^{2/3} \pi^{2/3} m^{4/3}} \right) + F \times \left(\frac{n^2 h^2}{4\pi^2 mF} \right)^{1/3}$$

$$\Rightarrow E = \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3} \left[\frac{1}{2} + 1 \right]$$

$$= \frac{3}{2} \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3}$$

8. The filament of a light bulb has surface area 64 mm^2 . The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m . Assume the pupil of the eyes of the observer to be circular with radius 3 mm . Then

(Take Stefan-Boltzmann constant $= 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$, Wien's displacement constant $= 2.90 \times 10^{-3} \text{ m-K}$, Planck's constant $= 6.63 \times 10^{-34} \text{ Js}$, speed of light in vacuum $= 3.00 \times 10^8 \text{ ms}^{-1}$)-

- (A) power radiated by the filament is in the range 642 W to 645 W
 (B) radiated power entering into one eye of the observer is in the range $3.15 \times 10^{-8} \text{ W}$ to $3.25 \times 10^{-8} \text{ W}$
 (C) the wavelength corresponding to the maximum intensity of light is 1160 nm
 (D) taking the average wavelength of emitted radiation to be 1740 nm , the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

Ans. (B,C,D)

Sol. $A = 64 \text{ mm}^2$, $T = 2500 \text{ K}$ (A = surface area of filament, T = temperature of filament, d is distance of bulb from observer, R_e = radius of pupil of eye)

Point source $d = 100 \text{ m}$

$$R_e = 3 \text{ mm}$$

$$\begin{aligned} \text{(A)} \quad P &= \sigma A e T^4 \\ &= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4 \quad (e = 1 \text{ black body}) \\ &= 141.75 \text{ W} \end{aligned}$$

Option (A) is wrong

(B) Power reaching to the eye

$$\begin{aligned} &= \frac{P}{4\pi d^2} \times (\pi R_e^2) \\ &= \frac{141.75}{4\pi \times (100)^2} \times \pi \times (3 \times 10^{-3})^2 \\ &= 3.189375 \times 10^{-8} \text{ W} \end{aligned}$$

Option (B) is correct

(C) $\lambda_m T = b$

$$\begin{aligned} \lambda_m \times 2500 &= 2.9 \times 10^{-3} \\ \Rightarrow \lambda_m &= 1.16 \times 10^{-6} \\ &= 1160 \text{ nm} \end{aligned}$$

Option (C) is correct

$$\text{(D)} \quad \text{Power received by one eye of observer} = \left(\frac{hc}{\lambda} \right) \times \dot{N}$$

\dot{N} = Number of photons entering into eye per second

$$\begin{aligned} \Rightarrow 3.189375 \times 10^{-8} &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1740 \times 10^{-9}} \times \dot{N} \end{aligned}$$

$$\Rightarrow \dot{N} = 2.79 \times 10^{11}$$

Option (D) is correct

9. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows: [position] = $[X^\alpha]$; [speed] = $[X^\beta]$; [acceleration] = $[X^p]$; [linear momentum] = $[X^q]$; [force] = $[X^r]$. Then -

$$\text{(A)} \quad \alpha + p = 2\beta$$

$$\text{(B)} \quad p + q - r = \beta$$

$$\text{(C)} \quad p - q + r = \alpha$$

$$\text{(D)} \quad p + q + r = \beta$$

Ans. (A,B)

Sol. Given $L = x^\alpha$ (1)

$LT^{-1} = x^\beta$ (2)

$LT^{-2} = x^p$ (3)

$MLT^{-1} = x^q$ (4)

$MLT^{-2} = x^r$ (5)

$$\frac{(1)}{(2)} \Rightarrow T = x^{\alpha-\beta}$$

From (3)

$$\frac{x^\alpha}{x^{2(\alpha-\beta)}} = x^p$$

$$\Rightarrow \alpha + p = 2\beta \quad (A)$$

From (4)

$$M = x^{q-\beta}$$

From (5) $\Rightarrow x^q = x^r x^{\alpha-\beta}$

$$\Rightarrow \alpha + r - q = \beta \quad \dots\dots(6)$$

Replacing value ' α ' in equation (6) from (A)

$$2\beta - p + r - q = \beta$$

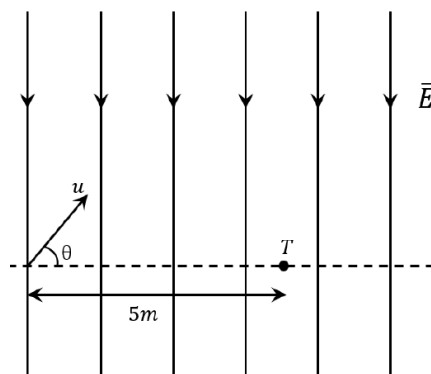
$$\Rightarrow p + q - r = \beta \quad (B)$$

Replacing value of ' β ' in equation (6) from (A)

$$2\alpha + 2r - 2q = \alpha + p$$

$$\alpha = p + 2q - 2r$$

- 10.** A uniform electric field, $\vec{E} = -400\sqrt{3}\hat{y} \text{ NC}^{-1}$ is applied in a region. A charged particle of mass m carrying positive charge q is projected in this region with an initial speed of $2\sqrt{10} \times 10^6 \text{ ms}^{-1}$. This particle is aimed to hit a target T , which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10} \text{ Ckg}^{-1}$. Then-



- (A) the particle will hit T if projected at an angle 45° from the horizontal
- (B) the particle will hit T if projected either at an angle 30° or 60° from the horizontal
- (C) time taken by the particle to hit T could be $\sqrt{\frac{5}{6}} \mu\text{s}$ as well as $\sqrt{\frac{5}{2}} \mu\text{s}$
- (D) time taken by the particle to hit T is $\sqrt{\frac{5}{3}} \mu\text{s}$

Ans. (B,C)

Sol. $a_y = -400\sqrt{3} \times 10^{10} [qE_y = ma_y]$

$$R = 5 = \frac{40 \times 10^{12} \sin 2\theta}{400\sqrt{3} \times 10^{10}} \left[R(\text{range}) = \frac{u^2 \sin 2\theta}{a_y} \right]$$

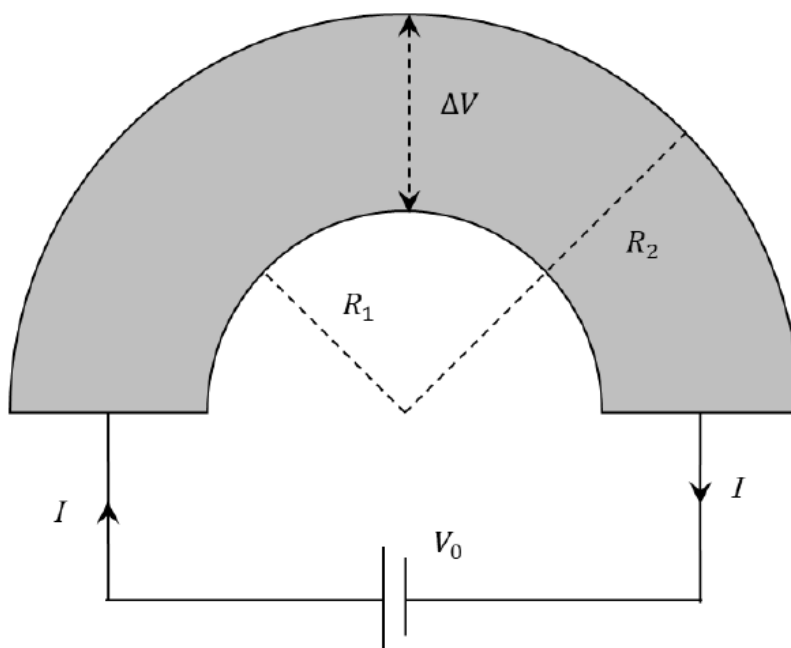
$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = 60^\circ, 120^\circ \Rightarrow \theta = 30^\circ, 60^\circ$$

$$\text{Time of flight } T_1 = \frac{2 \times 2\sqrt{10} \times 10^6 \times \frac{1}{2}}{400\sqrt{3} \times 10^{10}} = \sqrt{\frac{5}{6}} \mu\text{s} \quad (\text{for } \theta = 30^\circ)$$

$$\text{Time of flight } T_2 = \frac{2 \times 2\sqrt{10} \times 10^6 \times \frac{\sqrt{3}}{2}}{400\sqrt{3} \times 10^{10}} = \sqrt{\frac{5}{2}} \mu\text{s} \quad (\text{for } \theta = 60^\circ)$$

- 11.** Shown in the figure is a semicircular metallic strip that has thickness t and resistivity ρ . Its inner radius is R_1 and outer radius is R_2 . If a voltage V_0 is applied between its two ends, a current I flows in it. In addition, it is observed that a transverse voltage ΔV develops between its inner and outer surfaces due to purely kinetic effects of moving electrons (ignore any role of the magnetic field due to the current). Then (figure is schematic and not drawn to scale)-



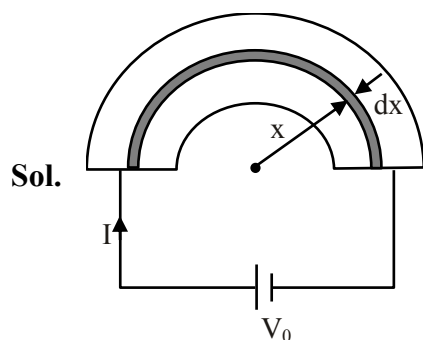
(A) $I = \frac{V_0 t}{\pi \rho} \ln\left(\frac{R_2}{R_1}\right)$

(B) the outer surface is at a higher voltage than the inner surface

(C) the outer surface is at a lower voltage than the inner surface

(D) $\Delta V \propto I^2$

Ans. (A,C,D)



All the elements are in parallel

$$\therefore \int \frac{1}{dr} = \int_{R_1}^{R_2} \frac{t dx}{\rho \pi x}$$

$$\frac{1}{r} = \frac{t}{\pi \rho} \ln\left(\frac{R_2}{R_1}\right)$$

$$\text{Resistance} = \frac{\pi \rho}{t \ln\left(\frac{R_2}{R_1}\right)}$$

$$i = \frac{V_0 t \ln\left(\frac{R_2}{R_1}\right)}{\pi \rho} \quad (A)$$

$(-e\vec{E})$ will be inward direction in order to provide centripetal acceleration. Therefore electric field will be radially outward

$$V_{\text{outer}} < V_{\text{inner}} \quad (C)$$

$$\frac{mV_d^2}{r} = q\vec{E}$$

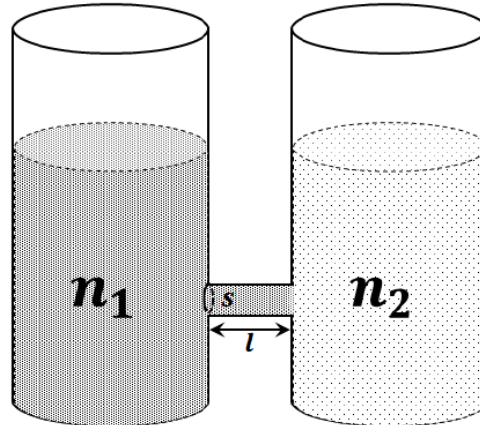
$$E = \frac{mV_d^2}{qr} \quad (I = neAV_d \Rightarrow V_d \propto i)$$

$$\Delta V = \int \vec{E} \cdot d\vec{r}$$

$$\Delta V \propto V_d^2$$

$$\Delta V \propto I^2$$

12. As shown schematically in the figure, two vessels contain water solutions (at temperature T) of potassium permanganate (KMnO_4) of different concentrations n_1 and n_2 ($n_1 > n_2$) molecules per unit volume with $\Delta n = (n_1 - n_2) \ll n_1$. When they are connected by a tube of small length ℓ and cross-sectional area S , KMnO_4 starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed v of the molecules is limited by the viscous force $-\beta v$ on each molecule, where β is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? (k_B is the Boltzmann constant)-



(A) the force causing the molecules to move across the tube is $\Delta n k_B T S$

(B) force balance implies $n_1 \beta v \ell = \Delta n k_B T$

(C) total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{\ell}\right) \left(\frac{k_B T}{\beta}\right) S$

(D) rate of molecules getting transferred through the tube does not change with time

Ans. (A,B,C)

Sol. $n_1 \gg (n_1 - n_2) = \Delta n$

$$p_1 = \frac{n_1 RT}{N_A} \quad p_2 = \frac{n_2 RT}{N_A}$$

$$F = (n_1 - n_2) k_B T S = \Delta n k_B T S \quad (A)$$

$$V = \frac{\Delta n k_B T S}{\beta}$$

Force balance \Rightarrow Pressure \times Area = Total number of molecules $\times \beta v$

$$\Delta n k_B T S = \ell n_1 S \beta v$$

$$\Rightarrow n_1 \beta v \ell = \Delta n k_B T \quad (B)$$

$$\text{Total number of molecules/sec} = \frac{(n_1 v dt) S}{dt}$$

$$= n_1 v S = \frac{\Delta n k_B T v S}{\beta v \ell}$$

$$= \left(\frac{\Delta n}{\ell} \right) \left(\frac{k_B T}{\beta} \right) S \quad (C)$$

As Δn will decrease with time therefore rate of molecules getting transfer decreases with time.

SECTION-3 : (Maximum Marks : 24)

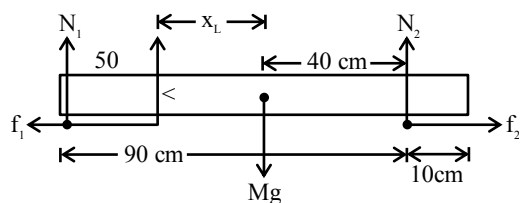
- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

<i>Full Marks</i>	: +4 If ONLY the correct numerical value is entered;
<i>Zero Marks</i>	: 0 In all other cases.

- 13.** Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is _____.

Ans. (25.60)

Sol.



Initially

$$\begin{array}{l|l}
 N_1 + N_2 = Mg & N_1 = \frac{4Mg}{9} \\
 (\tau_n = 0)_{\text{about centre}} N_1(50) = N_2(40) & N_2 = \frac{5Mg}{9} \\
 \hline
 5N_1 = 4N_2 & \\
 \hline
 f_{1K} = \mu_K N_1 & f_{1L} = \mu_S N_1 \\
 f_{1K} = 0.32 N_1 & f_{1L} = 0.4 N_1 \\
 \hline
 f_{2K} = 0.32 N_2 & f_{2L} = 0.4 N_2
 \end{array}$$

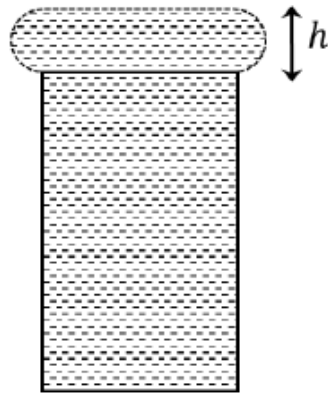
Suppose x_L = distance of left finger from centre when right finger starts moving

$$\begin{aligned}
 (\tau_n = 0)_{\text{about centre}} &\Rightarrow N_1 x_L = N_2(40) \\
 f_{K1} = f_{L2} &\Rightarrow 0.32 N_1 = 0.40 N_2 \\
 4N_1 &= 5N_2 \\
 N_1 x_L &= \frac{4N_1}{5} (40) \\
 x_L &= 32
 \end{aligned}$$

Now x_R = distance when right finger stops and left finger starts moving

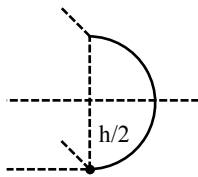
$$\begin{aligned}
 (\tau_n = 0)_{\text{about centre}} &\Rightarrow N_1 x_L = N_2(x_R) \\
 f_{L1} = f_{K2} &\Rightarrow 0.4 N_1 = 0.32 N_2 \\
 5N_1 &= 4N_2 \\
 \frac{4N_2}{5}(32) &= N_2 x_R \\
 x_R &= \frac{128}{5} = 25.6 \text{ cm}
 \end{aligned}$$

- 14.** When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kg m^{-3} , 0.07 Nm^{-1} and 10 ms^{-2} , respectively, the value of h (in mm) is _____.



Ans. (3.74)

Sol.



$$r = h/2$$

Pressure at the bottom of disc = pressure due to surface tension

$$\rho gh = T \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$R_1 \gg R_2$$

$$\text{So } \frac{1}{R_1} \ll \frac{1}{R_2} \text{ and } R_2 = h/2$$

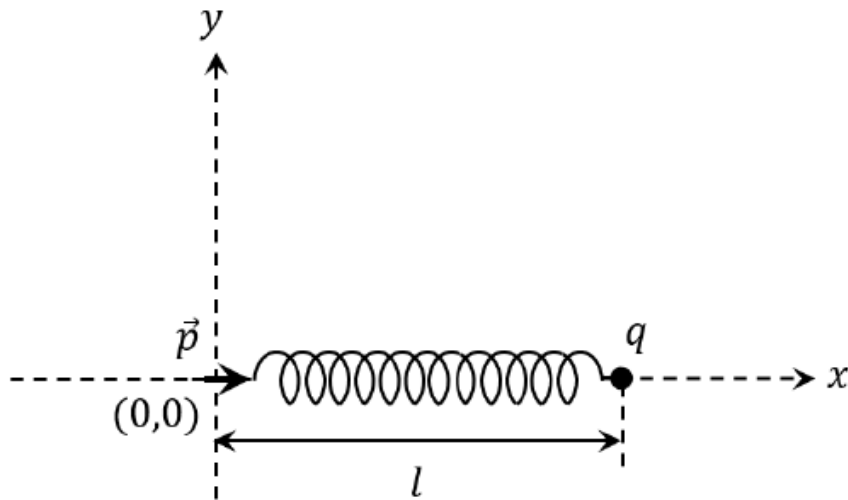
$$\therefore \rho gh = T \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = T \left[0 + \frac{1}{h/2} \right]$$

$$h^2 = \frac{2T}{\rho g}$$

$$h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{2 \times 0.07}{10^3 \times 10}} = \sqrt{\frac{14 \times 100}{10^4 \times 100}}$$

$$h = \sqrt{14} \text{ mm} = 3.741$$

- 15.** One end of a spring of negligible unstretched length and spring constant k is fixed at the origin $(0,0)$. A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole \vec{p} pointing towards the charge q is fixed at the origin, the spring gets stretched to a length ℓ and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta \ell \ll \ell$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta} \sqrt{\frac{k}{m}}$. The value of δ is _____.

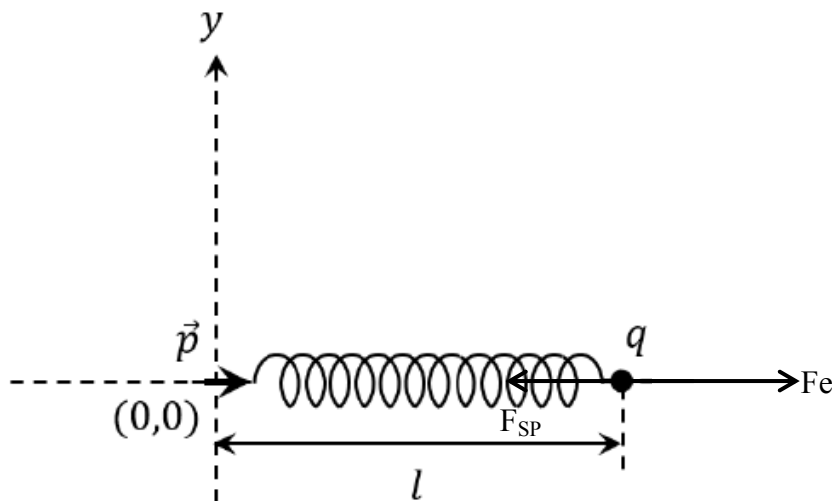


Ans. (3.14)

Sol. $\Delta \ell \rightarrow x$

At ℓ : $F_e = F_{SP}$

$$k\ell = \frac{2kpq}{\ell^3}$$



$$F_{\text{net}} = F_{\text{sp}} - F_e = k(\ell + x) - \frac{q(2kp)}{(\ell + x)^3}$$

$$= k(x + \ell) - \frac{q(2kp)}{\ell^3(1 + x/\ell)^3}$$

$$= kx + k\ell - q\left(\frac{2kp}{\ell^3}\right)\left(1 - \frac{3x}{\ell}\right)$$

$$= kx + k\ell - q\left(\frac{2kp}{\ell^3}\right) + \frac{2kpq}{\ell^3} \cdot \frac{3x}{\ell}$$

$$F_N = kx + k\ell \left(\frac{3x}{\ell} \right) = 4kx$$

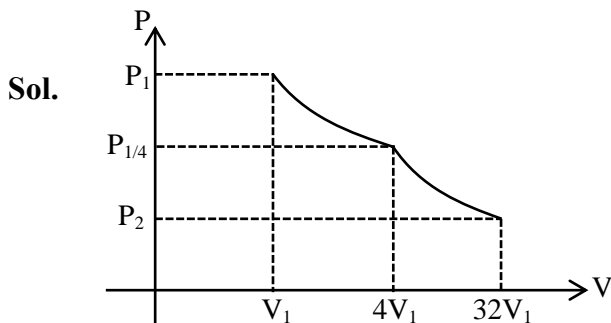
$$k_{eq} = 4k \quad T = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}}$$

$$\text{So } \delta = \pi = 3.14$$

16. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1 . It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia} , respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then f is _____.

Ans. (1.77 to 1.78)



$$\frac{P_1}{4} (4V_1)^{5/3} = P_2 (32V_1)^{5/3}$$

$$P_2 = \frac{P_1}{4} \left(\frac{1}{8} \right)^{5/3} = \frac{P_1}{128}$$

$$W_{adi} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{P_1 V_1 - \frac{P_1}{128} (32V_1)}{\frac{5}{3} - 1}$$

$$= \frac{P_1 V_1 (3/4)}{2/3} = \frac{9}{8} P_1 V_1$$

$$W_{iso} = P_1 V_1 \ln \left(\frac{4V_1}{V_1} \right) = 2P_1 V_1 \ln 2$$

$$\frac{W_{\text{iso}}}{W_{\text{adio}}} = \frac{2P_1 V_1 \ln 2}{\frac{9}{8} P_1 V_1} = \frac{16}{9} \ln 2 = f \ln 2$$

$$f = \frac{16}{9} = 1.7778 \approx 1.78$$

17. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1} , the smallest value of the percentage change required in the length of the pipe is _____.

Ans. (0.62 to 0.63)

Sol. $f \propto \frac{1}{\ell_1} \Rightarrow f = \frac{k}{\ell_1} \dots (1)$

($\ell_1 \Rightarrow$ initial length of pipe)

$$\left(\frac{V}{V - V_T} \right) f = \frac{k}{\ell_2} \quad \{V_T \text{ Speed of tuning fork, } \ell_2 \rightarrow \text{new length of pipe}\} \dots (2)$$

$$(1) \div (2)$$

$$\frac{V - V_T}{V} = \frac{\ell_2}{\ell_1}$$

$$\frac{\ell_2}{\ell_1} - 1 = \frac{V - V_T}{V} - 1$$

$$\frac{\ell_2 - \ell_1}{\ell_1} = \frac{-V_T}{V}$$

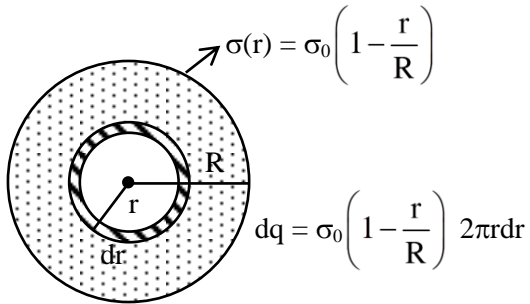
$$\frac{\ell_2 - \ell_1}{\ell_1} \times 100 = \frac{-2}{320} \times 100 = -0.625$$

Therefore smallest value of percentage change required in the length of pipe is 0.625

18. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0 \left(1 - \frac{r}{R} \right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0 . Electric flux through another spherical surface of radius $\frac{R}{4}$ and concentric with the disc is ϕ . Then the ratio $\frac{\phi_0}{\phi}$ is _____.

Ans. (6.40)

Sol.



$$\phi_0 = \frac{\int dq}{\epsilon_0} = \frac{\int_0^R \sigma_0 \left(1 - \frac{r}{R}\right) 2\pi r dr}{\epsilon_0}$$

$$\phi = \frac{\int dq}{\epsilon_0} = \frac{\int_0^{R/4} \sigma_0 \left(1 - \frac{r}{R}\right) 2\pi r dr}{\epsilon_0}$$

$$\therefore \frac{\phi_0}{\phi} = \frac{\sigma_0 2\pi \int_0^R \left(r - \frac{r^2}{R}\right) dr}{\sigma_0 2\pi \int_0^{R/4} \left(r - \frac{r^2}{R}\right) dr}$$

$$= \frac{\frac{R^2}{2} - \frac{R^2}{3}}{\frac{R^2}{32} - \frac{R^2}{3 \times 64}} = \frac{32}{5}$$

$$= 6.40$$