FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

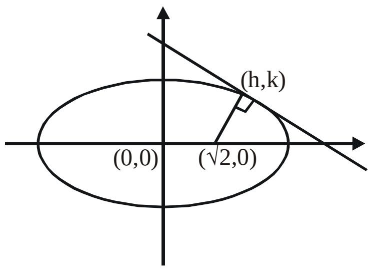
1. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $\left(-1,\sqrt{3}\right)$ (2) $\left(-1,\sqrt{2}\right)$

(3) $\left(-2,\sqrt{3}\right)$ (4) $(1,2)$

Official Ans. by NTA (1)

Sol. Let foot of perpendicular is (h,k)



$\frac{x^2}{4}+\frac{y^2}{2}=1$ (Given)

$a=2$, $b=\sqrt{2}$, $e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus $(ae,0)=\left(\sqrt{2},0\right)$

Equation of tangent

$y=mx+\sqrt{a^2m^2+b^2}$

$y=mx+\sqrt{4m^2+2}$

Passes throguh (h,k)

$(k-mh)^2=4m^2+2$ ...(1)

line perpendicular to tangent will have slope $-\frac{1}{m}$

$y-0=-\frac{1}{m}\left(x-\sqrt{2}\right)$

$my=-x+\sqrt{2}$

$(h+mk)^2=2$ ...(2)

Add equaiton (1) and (2)

$k^2(1+m^2)+h^2(1+m^2)=4(1+m^2)$

$h^2+k^2=4$

$x^2+y^2=4$ (Auxilary circle)

$\therefore$ $\left(-1,\sqrt{3}\right)$ lies on the locus.

2. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

Official Ans. by NTA (2)

Sol. Total numbers in three familes = 3 + 3 + 4 = 10

so total arrangement = 10!

Family 1	Family 2	Family 3
3	3	4

Favourable cases

$=\underset{\text{Arrangement of 3 Families}}{3!}\quad\underset{\text{Interval Arrangment of families members}}{3!\times3!\times4!}$

$\therefore$ Probability of same family memebers are together $=\frac{3!\,3!\,3!\,4!}{10!}=\frac{1}{700}$

so option(2) is correct.

3. $\lim_{x\to1}\left(\frac{\int_0^{(x-1)^2}t\cos\left(t^2\right)dt}{(x-1)\sin(x-1)}\right)$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

Official Ans. by NTA (1)

Official Ans. by ALLEN

(Bonus-Answers musbe zero)

Sol. $\lim\limits_{x\to 1} \dfrac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)} \left(\dfrac{0}{0}\right)$

Apply L Hopital Rule

$$= \lim_{x\to 1} \frac{2(x-1).(x-1)^2\cos(x-1)^4 - 0}{(x-1).\cos(x-1)+\sin(x-1)} \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$= \lim_{x\to 1} \frac{2(x-1)^2\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$= \lim_{x\to 1} \frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$= \frac{0}{1+1} = 0$$

4. If {p} denotes the fractional part of the number p, then $\left\{\dfrac{3^{200}}{8}\right\}$, is equal to

(1) $\dfrac{1}{8}$ (2) $\dfrac{5}{8}$ (3) $\dfrac{3}{8}$ (4) $\dfrac{7}{8}$

Official Ans. by NTA (1)

Sol. $\left\{\dfrac{3^{200}}{8}\right\} = \left\{\dfrac{(3^2)^{100}}{8}\right\}$

$$= \left\{\frac{(1+8)^{100}}{8}\right\}$$

$$= \left\{\frac{1+{}^{100}C_1.8+{}^{100}C_2.8^2+\ldots+{}^{100}C_{100}8^{100}}{8}\right\}$$

$$= \left\{\frac{1+8m}{8}\right\}$$

$$= \frac{1}{8}$$

5. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

Official Ans. by NTA (4)

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

Now $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

Now $D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

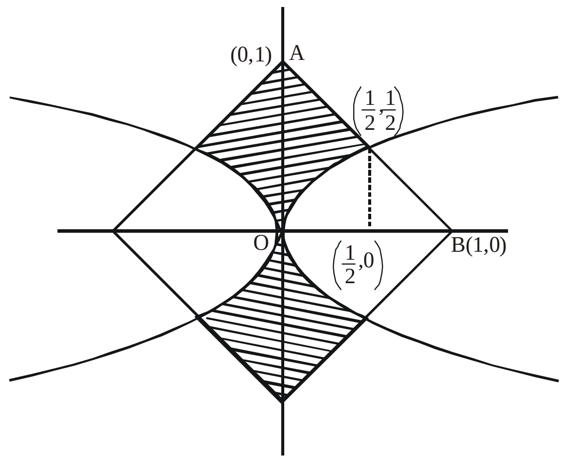
6. The area (in sq. units) of the region $A = \{(x,y) : |x| + |y| \le 1,\ 2y^2 \ge |x|\}$ is :

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{3}$ (3) $\dfrac{7}{6}$ (4) $\dfrac{5}{6}$

Official Ans. by NTA (4)

Sol. $|x| + |y| \le 1$

$2y^2 \ge |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = \frac{1}{2}$ or -1

Now Area of $\Delta OAB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of Region $R_1 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Area of Region $R_2 = \frac{1}{\sqrt{2}} \int_0^{\frac{1}{2}} \sqrt{x}\, dx = \frac{1}{6}$

Now area of shaded region in first quadrant

= Area of $\Delta OAB - R_1 - R_2$

$= \frac{1}{2} - \left(\frac{1}{6}\right) - \left(\frac{1}{8}\right) = \frac{5}{24}$

So required area $= 4\left(\frac{5}{24}\right) = \frac{5}{6}$

so option (4) is correct.

7. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$ (3) $\frac{5}{33}$ (4) $\frac{10}{99}$

Official Ans. by NTA (3)

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = $^{11}C_3$

Since these 3 numbers are in A.P. Let no's are a,b,c

2b $\Rightarrow$ even number

$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{pmatrix}$

so favourable cases = $^6C_2 + {}^5C_2$

= 15 + 10 = 25

P(3 numbers are in A.P. $= \frac{25}{^{11}C_3} = \frac{25}{165} = \frac{5}{33}$)

8. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

(1) $n\sqrt{a-1}$ (2) $\sqrt{a-1}$

(3) a – 1 (4) $\sqrt{n(a-1)}$

Official Ans. by NTA (2)

Sol. S.D $= \sqrt{\frac{\sum_{i=1}^{n}(x_i - a)}{n} - \left(\frac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2}$

$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2}$

{Given $\sum_{i=1}^{n}(x_i - a) = n$ $\sum_{i=1}^{n}(x_i - a)^2 = na$ }

$= \sqrt{a-1}$

9. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) x + 3 = 0 (2) x + 2y = 0

(3) 2x + 1 = 0 (4) x + 2 = 0

Official Ans. by NTA (1)

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y = m'(x+2) + \frac{2}{m'}$

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

since lines intersect at right angles

$\therefore mm' = -1$

Now $y = mx + m + \frac{1}{m}$ ...(1)

$$y = m'x + 2\left(m' + \frac{1}{m'}\right)$$

$$y = -\frac{1}{m}x + 2\left(-\frac{1}{m} - m\right)$$

$$y = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right) \quad ...(2)$$

From equation (1) and (2)

$$mx + m + \frac{1}{m} = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$$

$$\left(m + \frac{1}{m}\right)x + 3\left(m + \frac{1}{m}\right) = 0$$

$\therefore$ x + 3 = 0

10. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

Official Ans. by NTA (3)

Sol. Negation of $\phi \vee (\sim p \wedge q)$

$p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q)$

$= (T) \wedge (p \vee q)$

$= (p \vee q)$

now negation of $(p \vee q)$ is

$\sim (p \vee q) = \sim p \wedge \sim q$

11. If $f(x + y) = f(x)\, f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Official Ans. by NTA (2)

Sol. $f(x + y) = f(x).\, f(y)$

$\sum_{x=1}^{\infty} f(x) = 2$ where x,y $\in$ N

$f(1) + f(2) + f(3) + ... \infty = 2$ ...(1) (Given)

Now for $f(2)$ put x = y = 1

$f(2) = f(1 + 1) = f(1).\, f(1) = (f(1))^2$

$f(3) = f(2 + 1) = f(2).\, f(1) = (f(1))^3$

Now put these values in equation (1)

$f(1) + (f(1))^2 + [f(1)^2 + ...\infty = 2]$

$$\frac{f(1)}{1 - f(1)} = 2$$

$$\Rightarrow f(1) = \frac{2}{3}$$

Now $f(2) = \left(\frac{2}{3}\right)^2$

$$f(4) = \left(\frac{2}{3}\right)^4$$

then the value of $\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$

12. The general solution of the differential equation

$$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0 \text{ is :}$$

(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

Official Ans. by NTA (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2}+xy\dfrac{dy}{dx}=0$

$$\Rightarrow \sqrt{(1+x)^2(1+y^2)}+xy\frac{dy}{dx}=0$$

$$\Rightarrow \sqrt{1+x^2}\sqrt{1+y^2}=-xy\frac{dy}{dx}$$

$$\Rightarrow \int\frac{ydy}{\sqrt{1+y^2}}=-\int\frac{\sqrt{1+x^2}}{x}dx \quad ...(1)$$

Now put $1+x^2=u^2$ and $1+y^2=v^2$

$2xdx = 2udu$ and $2ydy = 2vdv$

$\Rightarrow xdx = udu$ and $ydy = vdv$

substitude these values in equation (1)

$$\int\frac{vdv}{v}=-\int\frac{u^2.du}{u^2-1}$$

$$\Rightarrow \int dv=-\int\frac{u^2-1+1}{u^2-1}du$$

$$\Rightarrow v=-\int\left(1+\frac{1}{u^2-1}\right)du$$

$$\Rightarrow v=-u-\frac{1}{2}\log_e\left|\frac{u-1}{u+1}\right|+c$$

$$\Rightarrow \sqrt{1+y^2}=-\sqrt{1+x^2}+\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$$

$$\Rightarrow \sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$$

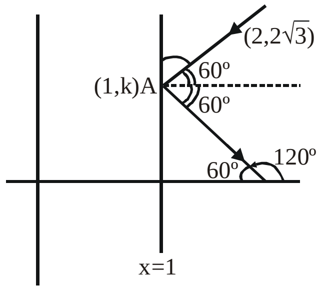
13. A ray of light coming from the point $\left(2,2\sqrt{3}\right)$ is incident at an angle 30° on the line x=l at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $\left(3,-\dfrac{1}{\sqrt{3}}\right)$ (2) $\left(3,-\sqrt{3}\right)$

(3) $\left(4,-\dfrac{\sqrt{3}}{2}\right)$ (4) $\left(4,-\sqrt{3}\right)$

Official Ans. by NTA (2)

Sol. For point A



$$\tan 60^\circ=\frac{2\sqrt{3}-k}{2-1}$$

$$\sqrt{3}=2\sqrt{3}-k$$

$$\therefore\ k=\sqrt{3}$$

so point $A\left(1,\sqrt{3}\right)$

Now slope of line AB is m_{AB} = tan120°

$$m\, m_{AB}=-\sqrt{3}$$

Now equation of line AB is

$$y-\sqrt{3}=-\sqrt{3}(x-1)$$

$$\sqrt{3}x+y=2\sqrt{3}$$

Now satisfy options

14. Let a,b,c,d and p be any non zero distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)=0$. Then :

(1) a,c,p are in G.P. (2) a,c,p are in A.P.

(3) a,b,c,d are in G.P. (4) a,b,c,d are in A.P.

Official Ans. by NTA (3)

Sol. $(a^2+b^2+c^2)p^2+2(ab+bc+cd)p+b^2+c^2+d^2=0$

$\Rightarrow (a^2p^2+2abp+b^2)+(b^2p^2+2bcp+c^2)+(c^2p^2+2cdp+d^2)=0$

$\Rightarrow (ab+b)^2+(bp+c)^2+(cp+d)^2=0$

This is possible only when

$ap+b=0$ and $bp+c=0$ and $cp+d=0$

$$p=-\frac{b}{a}=-\frac{c}{b}=-\frac{d}{c}$$

or $\dfrac{b}{a}=\dfrac{c}{b}=\dfrac{d}{c}$

$\therefore$ a,b,c,d are in G.P.

15. If $I_1=\int_0^1\left(1-x^{50}\right)^{100}dx$ and $I_2=\int_0^1\left(1-x^{50}\right)^{101}dx$ such that $I_2=\alpha I_1$ then α equals to

(1) $\dfrac{5050}{5051}$ (2) $\dfrac{5050}{5049}$

(3) $\dfrac{5049}{5050}$ (4) $\dfrac{5051}{5050}$

Official Ans. by NTA (1)

Sol. $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$

and $I_1 = \lambda I_2$

$$I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$$

$$I_2 = \int_0^1 \left(1-x^{50}\right)\left(1-x^{50}\right)^{100} dx$$

$$I_2 = \int_0^1 \left(1-x^{50}\right) dx - \int_0^1 x^{50}.\left(1-x^{50}\right)^{100} dx$$

$$I_2 = I_1 - \int_0^1 \underbrace{x}_{I}.\underbrace{x^{49}.\left(1-x^{50}\right)^{100} dx}_{II}$$

Now apply IBP

$$I_2 = I_1 - \left[x\int x^{49}.\left(1-x^{50}\right)^{100} dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.\left(1-x^{50}\right)^{100} dx\right]$$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101} dx\right]$$

$$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050}I_2$$

$$I_2 + \frac{1}{5050}I_2 = I_1 \Rightarrow \frac{5051}{5050}I_2 = I_1$$

$$\therefore\ \alpha = \frac{5050}{5051}$$

$$I_2 = \frac{5050}{5051}I_1$$

$\because I_2 = \alpha.I_1$

16. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$

(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

Official Ans. by NTA (4)

Sol. $\dfrac{f(t_2) - f(t_1)}{t_2 - t_1} = 2at + b$

$$\frac{a\left(t_2^2 - t_1^2\right) + b\left(t_2 - t_1\right)}{t_2 - t_1} = 2at + b$$

$\Rightarrow a(t_2 + t_1) + b = 2at + b$

$$\Rightarrow t = \frac{t_1 + t_2}{2}$$

17. The region represented by $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ is also given by the inequality :

(1) $y^2 \ge x + 1$ (2) $y^2 \ge 2(x + 1)$

(3) $y^2 \le x + \frac{1}{2}$ (4) $y^2 \le 2\left(x + \frac{1}{2}\right)$

Official Ans. by NTA (4)

Sol. $z = x + iy$

$|z| - ke(z) \le 1$

$$\Rightarrow \sqrt{x^2 + y^2} - x \le 1$$

$$\Rightarrow \sqrt{x^2 + y^2} \le 1 + x$$

$$\Rightarrow x^2 + y^2 \le 1 + 2x + x^2$$

$$\Rightarrow y^2 \le 2x + 1$$

$$\Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$$

18. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(1) 1 (2) 3

(3) 4 (4) 2

Official Ans. by NTA (4)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64$, $\alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{5/8}}$$

$$= \frac{64}{(256)^{5/8}}$$

$= 2$

19. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans. by NTA (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$x=\frac{-2z-4}{3}$ ...(3)

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$$\frac{3x+4}{-2}=-3y+1=z$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}}=\frac{y-\frac{1}{3}}{-\frac{1}{3}}=\frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}}=\frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}}=\frac{z-0}{1}$$

$$\text{S.D.}=\left|\frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}\right|$$

where $\vec{a}=(1,-1,0)$

$$\vec{b}=\left(-\frac{4}{3},\frac{1}{3},0\right)$$

$$\vec{c}=(0,-1,1)$$

$$\vec{d}=\left(-\frac{2}{3},-\frac{1}{3},1\right)$$

$$\Rightarrow \text{S.D}=\frac{1}{\sqrt{3}}$$

20. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}.$$

Then the ordered pair (m,M) is equal to

(1) (–3,–1) (2) (–4,–1)

(3) (1,3) (4) (–3,3)

Official Ans. by NTA (1)

Sol.

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$$=-1\left(\sin^2 x\right)-1\left(1+\sin 2x+\cos^2 x\right)$$

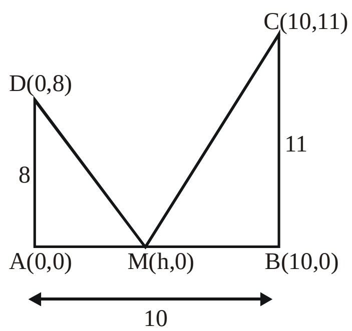
$$=-\sin 2x-2$$

$m = -3$, $M = -1$

21. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

Official Ans. by NTA (5.00)

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

$= 2(h^2 - 10h) + 285$

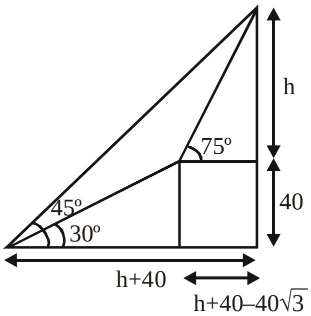
$= 2(h - 5)^2 + 235$

it is minimum if h = 5

22. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is_.

Official Ans. by NTA (80.00)

Sol.



$\tan 75^\circ = \dfrac{h}{h+40-40\sqrt{3}}$

$\dfrac{2+\sqrt{3}}{1} = \dfrac{h}{h+40-40\sqrt{3}}$

$\Rightarrow 2h + 80 - 80\sqrt{3} + \sqrt{3}h + 40\sqrt{3} - 120 = h$

$\Rightarrow h\left(\sqrt{3}+1\right) = 40 + 40\sqrt{3}$

$\Rightarrow h = 40$

$\therefore$ Height of hill = 40 + 40 = 80m

23. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _.

Official Ans. by NTA (28.00)

Sol. $2^m - 2^n = 112$

m = 7, n = 4

$(2^7 - 2^4 = 112)$

$m \times n = 7 \times 4 = 28$

24. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}|$ is _.

Official Ans. by NTA (4.00)

Sol. $\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}|$

$= \sqrt{3}\left(\sqrt{2+2\cos\theta}\right) + \sqrt{2-2\cos\theta}$

$= \sqrt{6}\left(\sqrt{1+\cos\theta}\right) + \sqrt{2}\left(\sqrt{1-\cos\theta}\right)$

$= 2\sqrt{3}\left|\cos\dfrac{\theta}{2}\right| + 2\left|\sin\dfrac{\theta}{2}\right|$

$\le \sqrt{\left(2\sqrt{3}\right)^2 + (2)^2} = 4$

25. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\dfrac{1}{x}\right) + 5x^2 & , \quad x < 0 \\ 0 & , \quad x = 0 \\ x^5 \cos\left(\dfrac{1}{x}\right) + \lambda x^2 & , \quad x > 0 \end{cases}$$

. The value of λ for which $f''(0)$ exists, is _.

Official Ans. by NTA (5.00)

Sol. $f(x) = x^5.\sin\dfrac{1}{x} + 5x^2$ if x < 0

$f(x) = 0$ if x = 0

$f(x) = x^5.\cos\dfrac{1}{x} + \lambda x^2$ if x > 0

LHD of $f'(x)$ at x = 0 is 10

RHD of $f'(x)$ at x = 0 is 2λ

if $f''(0)$ exists then

$2\lambda = 10$

$\Rightarrow \lambda = 5$