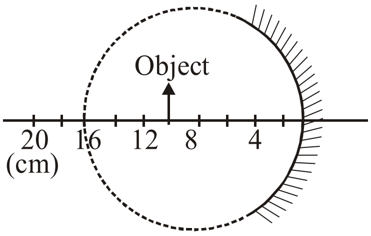
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1.



A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object ? (Figure drawn as schematic and not to scale)

(1) Inverted, real and magnified

(2) Erect, virtual and magnified

(3) Erect, virtual and unmagnified

(4) Inverted, real and unmagnified

Official Ans. by NTA (1)
Official Ans. by ALLEN (4)

Sol. $f = \frac{-8}{2} = -4\text{cm}$

u = –10 cm
v = ?

as $\frac{1}{v}\quad\frac{1}{u}\quad\frac{1}{f}$

$\frac{1}{v}\quad\left(\frac{1}{-10}\right)\quad\frac{1}{-4}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{4}$

$\frac{1}{v}\quad\frac{4-10}{40}$

$v\quad\frac{40}{-6}$

$v = \frac{-20}{3}$

$m\quad\frac{-v}{u}$

$m = \frac{-\left(-\frac{20}{3}\right)}{-10} \Rightarrow m = \frac{-2}{3}$

or image will be real, inverted and unmagnified.

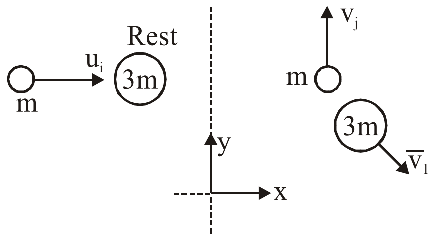
2. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by :

(1) $v = \sqrt{\frac{2}{3}}u$ (2) $v = \frac{1}{\sqrt{6}}u$

(3) $v\quad\frac{u}{\sqrt{3}}$ (4) $v = \frac{u}{\sqrt{2}}$

Official Ans. by NTA (4)

Sol.



From momentum conservation

$\vec{P}_i = \vec{P}_f$

m(ui) + 3m(0) = mvj + 3m$\overline{v}_1$

mui – mvj = 3m$\overline{v}_1$

$\overline{v}_1 = \frac{ui - vj}{3}$

or $|v_1| = \frac{\sqrt{u^2 + v^2}}{3}$

or $v_1^2 = \frac{u^2 + v^2}{9}$ ....(1)

As collision is perfectely elastic hence

$k_i = k_j$

$\frac{1}{2}mu^2 + \frac{1}{2}3m0^2 = \frac{1}{2}mv^2 + \frac{1}{2}3mv_1^2$

$\Rightarrow u^2 = v^2 + 3v_1^2$

$$u^2 = v^2 + 3\frac{(u^2+v^2)}{9}$$

$\Rightarrow 3u^2 = 3v^2 + u^2 + v^2$

$\Rightarrow 2u^2 = 4v^2$

$$v = \frac{u}{\sqrt{2}}$$

3. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to: (Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C

(1) 12 cm (2) 4 cm

(3) 5 cm (4) 2 cm

Official Ans. by NTA (2)

Sol. Pitch = $\frac{2\pi m}{qB} v\cos\theta$

$$\text{Pitch} = \frac{2(3.14)(1.67\times10^{-27})\times4\times10^5\times\cos60}{(1.69\times10^{-19})(0.3)}$$

Pitch = 0.04m = 4 cm

4. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity $\rho_C > \rho_T > \rho_M$ and ρ_A respectively. Then:

(1) $\rho_A > \rho_T > \rho_C$ (2) $\rho_C > \rho_A > \rho_T$

(3) $\rho_A > \rho_M > \rho_C$ (4) $\rho_M > \rho_A > \rho_C$

Official Ans. by NTA (4)

Sol. $\rho_M > \rho_A > \rho_C$

5. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?

(1) T : Large retentivity, small coercivity

(2) P : Small retentivity, large coercivity

(3) T : Large retentivity, large coercivity

(4) P : Large retentivity, large coercivity

Official Ans. by NTA (4)

Sol. As for permanent magnet large retentivity and large coercivity required

6. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is : (VSD is vernier scale division)

(1) 3.21 cm (2) 2.99 cm

(3) 3.2 cm (4) 3.07 cm

Official Ans. by NTA (4)

Sol. Least count = 1 mm or 0.01 cm

Zero error = 0 + 0.01 × 7 = 0.07 cm

Reading = 3.1 + (0.01 × 4) – 0.07

= 3.1 + 0.04 – 0.07

= 3.1 – 0.03

= 3.07 cm

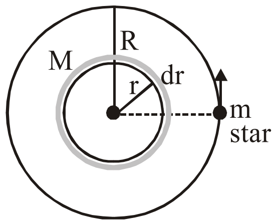
7. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as :

(1) $T \propto R$ (2) $T^2 \propto \frac{1}{R^3}$

(3) $T^2 \propto R$ (4) $T^2 \propto R^3$

Official Ans. by NTA (3)

Sol.



$dm = \rho dv$

$$dm = \left(\frac{k}{r}\right)(4\pi r^2 dr)$$

$dm = 4\pi k r dr$

$$M = \int_0^R dm = \int_0^R 4\pi k r dr$$

$$M = 4\pi k \left.\frac{r^2}{2}\right|_0^R$$

$M = 2\pi k(R^2 - 0)$

$M = 2\pi k R^2$

for circular motion gravitational force will provide required centripital force or

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{G(2\pi k R^2)m}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{2\pi G k R}$$

Time period $T = \frac{2\pi R}{v}$

$$T = \frac{2\pi R}{\sqrt{2\pi G k R}} \propto \sqrt{R}$$

or $T^2 \propto R$

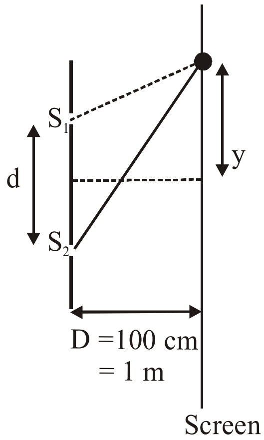
8. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to :

(1) 1.27 μm (2) 2 nm

(3) 2.87 nm (4) 2.05 μm

Official Ans. by NTA (1)

Sol.



$$y = \frac{nD\lambda}{d}$$

$$n = \frac{yd}{D\lambda} = \frac{1.27\times10^{-3}\times10^{-3}}{1\times632.8\times10^{-9}} = 2$$

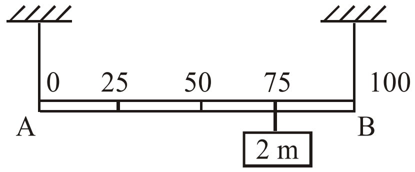
Path difference $\Delta x = n\lambda$

$= 2 \times 632.8$ nm

$= 1265.6$ nm

$= 1.27$ μm

9.



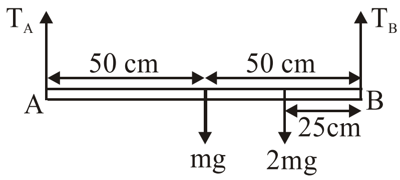
Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is :

(1) 2 mg (2) 0.5 mg

(3) 0.75 mg (4) 1 mg

Official Ans. by NTA (4)

Sol.



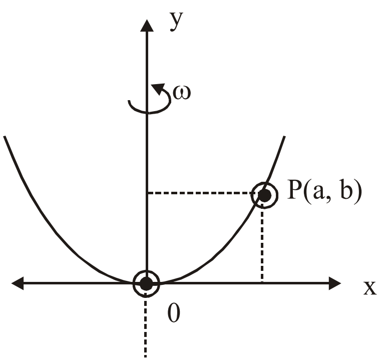
$\tau_B = 0$ (torque about point B is zero)

$(T_A) \times 100 - (mg) \times 50 - (2mg) \times 25 = 0$

$100\ T_A = 100$ mg

$T_A = 1$ mg

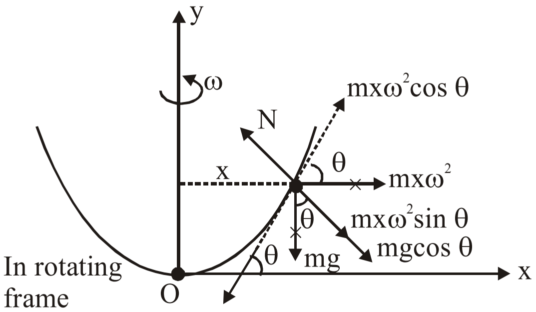
10. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) :



(1) $\sqrt{\frac{2gC}{ab}}$ (2) $2\sqrt{2gC}$ (3) $\sqrt{\frac{2g}{C}}$ (4) $2\sqrt{gC}$

Official Ans. by NTA (2)

Sol.



$mx\omega^2\cos\theta = mg\sin\theta$

$x\omega^2 = g\tan\theta$

$$x\omega^2 = g.\frac{dy}{dx}$$

$x\omega^2 = g.(8cx)$

$\omega^2 = 8gc$

$$\omega = 2\sqrt{2gc}$$

11. A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/ m^3 in vacuum. The amplitude of the magnetic field of the wave is close to

($\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{Nm^2}{C^2}$ and speed of light $= 3 \times 10^8$ ms^{-1}) :

(1) 180 nT (2) 160 nT

(3) 150 nT (4) 190 nT

Official Ans. by NTA (2)

Sol. Energy density $\frac{dU}{dV} = \frac{B_0^2}{2\mu_0}$

$$1.02\times10^{-8} = \frac{B_0^2}{2\times4\pi\times10^{-7}}$$

$$B_0^2 = (1.02\times10^{-8})\times(8\pi\times10^{-7})$$

$B_0 = 16 \times 10^{-8}$ T $= 160$ nT

12. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, $N = 6.023 \times 10^{26}$ per kilo mole and 1 eV $= 1.6 \times 10^{-19}$ J. The power output of the reactor is close to :

(1) 125 MW (2) 60 MW

(3) 35 MW (4) 54 MW

Official Ans. by NTA (2)

Sol. Number of uranium atoms in 2kg

$$= \frac{2\times6.023\times10^{26}}{235}$$

energy from one atom is 200×10^6 e.v. hence total energy from 2 kg uranium

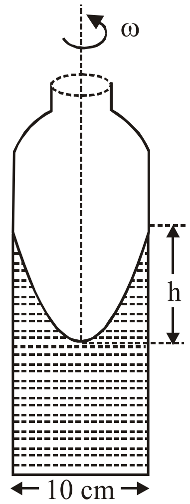
$$= \frac{2\times6.023\times10^{26}}{235}\times200\times10^6\times1.6\times10^{-19}\,J$$

2 kg uranium is used in 30 days hence this energy is recieved in 30 days hence energy recived per second or power is

$$\text{Power} = \frac{2\times6.023\times10^{26}\times200\times10^6\times1.6\times10^{-19}}{235\times30\times24\times3600}$$

Power $= 63.2 \times 10^6$ watt or 63.2 Mega Watt

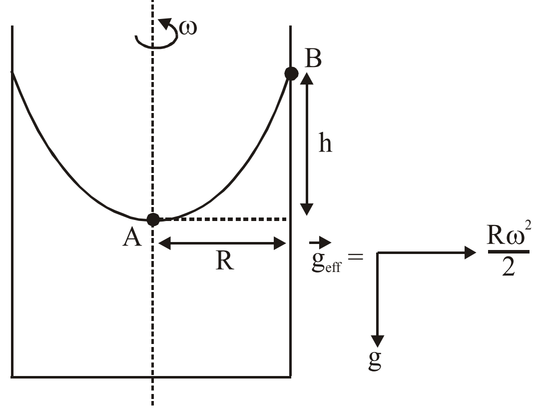
13. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h(in cm) of liquid at the centre of vessel and at the side will be:



(1) $\frac{25\omega^2}{2g}$ (2) $\frac{2\omega^2}{5g}$ (3) $\frac{5\omega^2}{2g}$ (4) $\frac{2\omega^2}{25g}$

Official Ans. by NTA (1)

Sol.



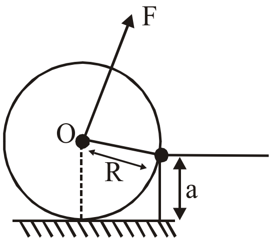
Applying pressure equation from A to B

$$P_0 + \rho.\frac{R\omega^2}{2}.R - \rho gh = P_0$$

$$\frac{\rho R^2\omega^2}{2} = \rho gh$$

$$h = \frac{R^2\omega^2}{2g} = (5)^2\frac{\omega^2}{2g} = \frac{25}{2}\frac{\omega^2}{g}$$

14. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :

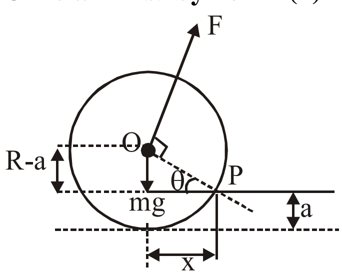


(1) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (2) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(3) $Mg\frac{a}{R}$ (4) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

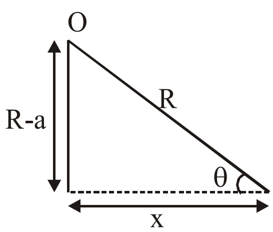
Official Ans. by NTA (4)

Sol.



$(\tau)_P = 0$

F.R. – mgx = 0



$$x = \sqrt{R^2-(R-a)^2}$$

$$F = mg\frac{x}{R}$$

$$F = mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$$

= minimum value of force to pull

15. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is :

(1) 11 (2) 15

(3) 20 (4) 13

Official Ans. by NTA (2)

Sol. $u = \frac{f_1 n_1 RT}{2} + \frac{f_2 n_2 RT}{2}$

$$u = \frac{5}{2}\times 3RT + \frac{3\times 5RT}{2} = 15RT$$

16. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be :

(1) $FA^{-1}V^0$ (2) FA^2V^{-1}

(3) FA^2V^{-3} (4) FA^2V^{-2}

Official Ans. by NTA (1)

Sol. $Y = F^xA^yV^z$

$M^1L^{-1}T^{-2} = [MLT^{-2}]^x[L^2]^y[LT^{-1}]^z$

$M^1L^1T^{-2} = [M]^x[L]^{x+2y+z}[T]^{-2x-z}$

comparing power of ML and T

x = 1...(1)

x + 2y + z = –1 ....(2)

–2x – z = –2 ...(3)

after solving

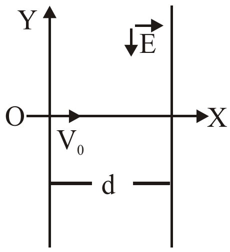
x = 1

y = –1

z = 0

$Y = FA^{-1}V^0$

17. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is :



(1) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$

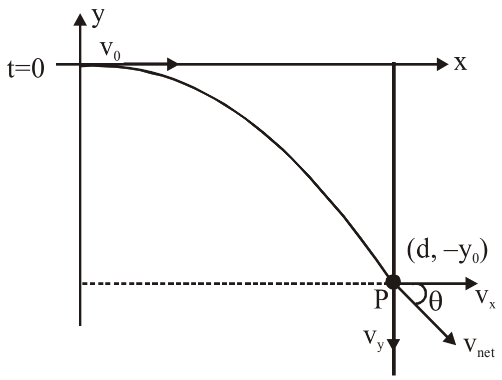
(2) $y = \frac{qEd}{mV_0^2}(x - d)$

(3) $y = \frac{qEd}{mV_0^2}x$

(4) $y = \frac{qEd^2}{mV_0^2}x$

Official Ans. by NTA (1)

Sol.



Let particle have charge q and mass 'm'

Solve for (q,m) mathematically

$F_x = 0$, $a_x = 0$, $(v)_x$ = constant

time taken to reach at 'P' = $\frac{d}{v_0} = t_0$ (let) ...(1)

(Along –y), $y_0 = 0 + \frac{1}{2}.\frac{qE}{m}.t_0^2$ ....(2)

$v_x = v_0$

$v = u + at$ (along -ve 'y')

speed $v_{y0} = \frac{qE}{m}.t_0$

$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m.v_0}, (t_0 = \frac{d}{v_0})$

$\tan\theta = \frac{qEd}{m.v_0^2}$

$\boxed{\text{slope} = \frac{-qEd}{mv_0^2}}$

Now we have to find eqⁿ of straight line whose slope is $\frac{-qEd}{mv_0^2}$ and it pass through point → (d, $-y_0$)

Because after x > d

No electric field ⇒ $F_{net} = 0$, $\vec{v}$ = const.

$y = mx + c$, $\left\{\begin{matrix} m = \frac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix}\right\}$

$-y_0 = \frac{-qEd}{mv_0^2}.d + c \Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$

Put the value

$y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2}$

$y_0 = \frac{1}{2}.\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$

$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$

$y = \frac{-qEd}{mv_0^2}x + \frac{1}{2}\frac{qEd^2}{mv_0^2}$

$\boxed{y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)}$

18. An amplitude modulated wave is represented by the expression $v_m = 5(1 + 0.6 \cos 6280t) \sin(211 \times 10^4 t)$ volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively :

(1) 5V, 8V (2) $\frac{3}{2}$V, 5V

(3) $\frac{5}{2}$V, 8V (4) 3V, 5V

Official Ans. by NTA (3)
Official Ans. by ALLEN
(Close Option is 3 Amax. = 8, Amin. = 2)

Sol. $V_m = 5(1+0.6 \cos 6280t) \sin(2\pi \times 10^4 t)$

$V_m = [5+3\cos 6820t] \sin(2\pi \times 10^4 t)$

$V_{max.} = 5 + 3 = 8$

$V_{min.} = 5 - 3 = 2$

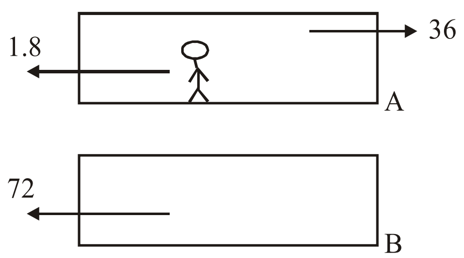
19. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hr. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1) 30.5 ms^{-1} (2) 29.5 ms^{-1}

(3) 31.5 ms^{-1} (4) 28.5 ms^{-1}

Official Ans. by NTA (2)

Sol.



Velocity of man with respect to ground

$\vec{V}_{m/g} = \vec{V}_{m/A} + \vec{V}_A = -1.8 + 36$

Velocity of man w.r.t. B

$\vec{V}_{m/B} = \vec{V}_m - \vec{V}_B$

$= -1.8 + 36 - (-72)$

$= 106.2$ km/hr

$= 29.5$ m/s

20. Two identical strings X and Z made of same material have tension T_X and T_Z in them. It their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X/T_Z is :

(1) 0.44 (2) 1.5

(3) 2.25 (4) 1.25

Official Ans. by NTA (3)

Sol. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

For identical string l and μ will be same

$f \propto \sqrt{T}$

$$\frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$$

$$\frac{T_x}{T_y} = \frac{9}{4} = 2.25$$

21. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$J then value of X to the nearest integer is________.

Official Ans. by NTA (36)
Official Ans. by ALLEN (4 Actual 4.033)

Sol. $u_i = \frac{1}{2} \times 5 \times 10^{-6} (220)^2$

Final common potential

$$v = \frac{220 \times 5 + 0 \times 2.5}{5 + 2.5} = 220 \times \frac{2}{3}$$

$$u_f = \frac{1}{2}(5+2.5) \times 10^{-6}\left(220 \times \frac{2}{3}\right)^2$$

$\Delta u = u_f - u_i$
$\Delta u = -403.33 \times 10^{-4}$

$\Rightarrow -403.33 \times 10^{-4} = \frac{X}{100}$

$X = -4.03$
or magnitude or value of X is approximate 4

22. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabaticaly to l/l0th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is ________.

Official Ans. by NTA (46)

Official Ans. by ALLEN (46 Actual 45.78)

Sol. Diatomic :

$f = 5$

$\gamma = 7/5$

$T_i = T = 273 + 20 = 293$ K

$V_i = V$

$V_f = V/10$

Adiabatic $\quad TV^{\gamma-1} = \text{constant}$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T.V^{7/5-1} = T_2\left(\frac{V}{10}\right)^{7/5-1}$$

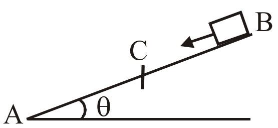
$\Rightarrow T_2 = T.\ 10^{2/5}$

$$\Delta U = \frac{nfR(T_2 - T_1)}{2} = \frac{5\times5\times\frac{25}{3}\times(T.10^{2/5} - T)}{2}$$

$$= \frac{25\times25\times}{6}\,T\,(10^{2/5} - 1)$$

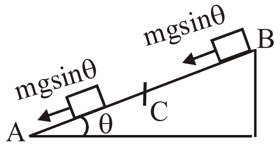
$$= \frac{625\times293\times(10^{2/5} - 1)}{6}$$

$$= 4.033\times10^3 \approx 4\text{kJ}$$

23. 

A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by μ = k tanθ). The value of k is ________.

Official Ans. by NTA (3)

Sol. 

Apply work energy theorem

$mg\sin\theta\ (AC + 2AC) - \mu mg\cos\theta AC = 0$

$\mu = 3\tan\theta$

24. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2s. The maximum value of EMF induced (in μV) in the coil will be close to the integer ________.

Official Ans. by NTA (15)

Sol. $r = 0.1$ m $\quad \frac{T}{2} = 0.2$ sec

$B = 3 \times 10^{-5}$ m $\qquad T = 0.4$ sec

At any time

flux $\phi = BA\cos\omega t$

$$|emf| = \left|\frac{d\phi}{dt}\right| = |BA\omega\ \sin\omega t|$$

$$(emf)_{max} = BA\omega = BA\ \frac{2\pi}{T}$$

$$= \frac{3\times10^{-5}\times\pi\times(0.1)^2\times2\pi}{0.4}$$

$$= \frac{6\pi^2}{4}\times10^{-6} \qquad \left(\begin{matrix}\pi^2 \simeq 10\\ \text{take}\end{matrix}\right)$$

$= 15 \times 10^{-6}$

$= 15\ \mu V$

25. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$. If the threshold wavelength for the metallic surface is nλ then value of n will be _______.

Official Ans. by NTA (9)

Sol. $\frac{hc}{\lambda}=\frac{hc}{\lambda_0}+eV$ .....(i)

$\frac{hc}{3\lambda}=\frac{hc}{\lambda_0}+\frac{e\cdot V}{4}$ .....(ii)

(multiply by 4)

$\frac{4hc}{3\lambda}=\frac{4hc}{\lambda_0}+eV$ ....(iii)

From (i) & (iii)

$$\frac{hc}{\lambda}-\frac{hc}{\lambda_0}=\frac{4hc}{3\lambda}-\frac{4hc}{\lambda_0}$$

$$-\frac{hc}{3\lambda}=-\frac{3hc}{\lambda_0}$$

$$\boxed{9\lambda=\lambda_0}$$

n = 9